ALTERNATING CURRENT

The instantaneous value of an AC is given by : $I = I_0 \sin \omega t$

and the alternaing voltage is given by : $E = E_0 \sin \omega t$

Here, ω is the angular frequency of Ac and $(\omega / 2\pi)$ is the frequency of Ac. $(2\pi / \omega)$ represents the time period of AC.The frequency of AC represents the number of cycles of AC completed in one second. AC supplied in India has a frequency of 50Hz.

Mean Value :

The mean value of AC represented by the equation, $I = I_0 \sin \omega t$, is zero over one complete cycle and is meaningless. In practice, mean value of alternating current refers to its average value over half cycle.

$$I_{mean} = \frac{2I_0}{\pi}$$

- A moving coil galvanometer, connected to an AC source of 50 Hz AC, shows a steady zero reading of the pointer. If the frequency is 2 Hz, the pointer oscillates with equal amplitude on either side of zero position.

  RMS or Virtual value : The RMS value of defined as the square root of the mean of square of the instantaneous value of current over the complete cycle. It may also be defined as the direct current which produces the same heating effect in a resistor as the actual AC in the same time.

$$I_V = \frac{I_0}{\sqrt{2}} = I_{rms}$$

- AC ammeter or voltmeter measures virtual current or virtual voltage (these are hot wire instrument)
- Form Factor $= \frac{\text{Virtual value}}{\text{Mean value}} = \frac{e_0/\sqrt{2}}{2e_0/\pi} = \frac{\pi}{2\sqrt{2}} = 1.1$

(A) AC through pure resistor R :

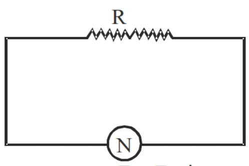
(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

(ii) A resistance oppose current but does not oppose a change in current. Hence, current is in phase with e.m.f.

(iii) The instantaneous value of the current is given by :

$$I = \frac{E_0 \sin \omega t}{R}$$

The virtual value of current I_V is given by $I_V = \frac{E_V}{R}$



(B) AC through pure inductor of inductance L :

(i) Alternating e.m.f. of the source : $E = E_0 \sin \omega t$

(ii) An inductor does not opposte current but opposes a change in current.

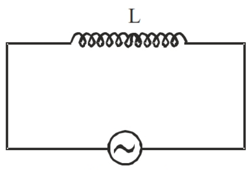
(iii) Since the voltage changes continuously, Hence, the current reacts to the change and inductive reactance

$$X_L = \omega L$$

(iv) The current lags behind the voltage by $\pi / 2$

(v) The instantaneous current is given by : $I = \left(\frac{E_0}{\omega L}\right) \sin\left(\omega t - \frac{\pi}{2}\right)$

The virtual value of the current is given by $I_V = E_V / \omega L$



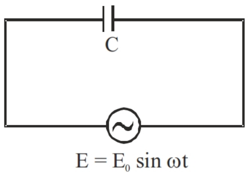
(C) AC through a pure capacitor of capacitance C :

(i) Alternating e.m.f. of the source. $E = E_0 \sin \omega t$.

(ii) A capacitor has infinite resistance for a DC source. With an AC source, voltage changes, hence charge ont he plates of the capacitor changes with time i.e. there is a current. The current leads the voltage by $\pi / 2$.

(iii) The capacitor has a capacitive reactang X_C in AC circuit, given by : $X_C = (1/\omega C)$

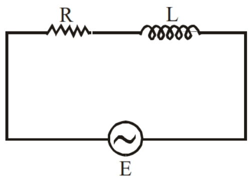
(iv) The instantaneous valeu of current is



$$I = \frac{E_0}{\left(\frac{1}{\omega C}\right)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

The virtual value of current I_V is given by : $I_V = \dfrac{E_V}{(1/\omega C)}$

(D) AC through RL circuit :

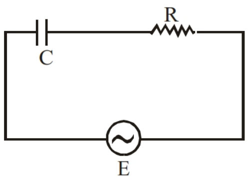


(i) In this case, we have impedance.

$$Z = \sqrt{R^2 + (X_L)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t - \phi)$$

(ii) Phase angle, $\tan \phi = (X_L / R) = (\omega L / R)$

(E) AC through CR circuit :

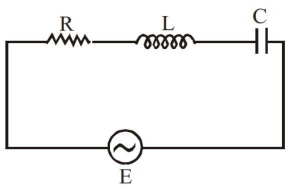


(i) In this case, we have impedence

$$Z = \sqrt{R^2 + (X_C)^2} \quad \text{and} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(ii) Phase angle, $\tan \phi = (X_C / R) = (I / \omega CR)$

(F) AC source connected to resistor R, an inductor L and a capacitor C in series :



(i) The virtual current I_V is given by :

$$I_V = \frac{E_V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

(ii) The current and voltage have a phase difference ϕ given by :

$$\tan \phi = \frac{\omega L - \frac{1}{\omega C}}{R} \quad \text{and So, current} \quad I = \frac{E_0}{z} \sin(\omega t + \phi)$$

(iii) The impedance which represents the effective resistance of the circuit to AC source is represented by Z. The impedance Z is the vector sum of resistance and reactances in AC series circuit.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

(iv) In AC series combination, virtual voltage are added vectorially i.e.

$$Ev = \sqrt{V_R^2 + (V_L - V_C)^2}$$

where $V_R = I_V R$, $V_C = I_V \, (1/C\omega)$ and $V_L = I_V (\omega L)$

Power in an AC circuit :

(i) The power in an electric circuit is the rate at which electric energy is consumed in the circuit.

Average power, $< P > = E_{rms} \times I_{rms} \times \cos\phi$

So, the product of rms value of voltage and current when multiplied by $\cos\phi$ gives the power dessipated.

(ii) $\cos\phi$ is known as power factor. For a LCR series circuit.

$$\cos\phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{R}{z}$$

(iii) Wattless current : we know that the average power in a circuit is given by :

$< P >= E_{rms} \times I_{rms} \times \cos\phi$; Here $(I_{rms} \cos\phi)$ is called watt less current which does not contribute in power dessipation.

Resonance in Series R-L-C circuit :

(i) A particular frequency of AC at which impedance of a series LCR circuit becomes minimum or the current becomes maximum is called the resonant frequency and the circuit is called as series resonance circuit.

(ii) At resonance frequency

$$\omega_0 L = \frac{1}{\omega_0 C} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

(iii) At resonance $\tan\phi = \dfrac{\omega_0 L - \frac{1}{\omega_0 C}}{R} = \text{zero}$ i.e. $\phi = 0$ i.e. phase difference is zero. So, $\cos\phi = +1$, i.e. power factor is maximum.

(iv) At resonance, $I_{max} = \dfrac{E_V}{Z_{min}} = \dfrac{E_V}{R}$

In this case, $V_L = I_V \omega_0 L$, $\quad V_C = I_V \; (1/\omega_0 C)$

$\because V_L = V_C$ and $E_V = \sqrt{V_R^2 + (V_L - V_C)^2}$ hence, $E_V = V_R$

i.e. at resonance with L and C in series, the current is maximum through the L and C combination but potential difference across the combination is zero.

(v) $Q = \dfrac{\omega L}{R}$ or $\dfrac{1}{\omega CR}$ i.e. the quality factor may be defined as the ratio of reactance of either inductance or capacitance at resonance frequency to the resistance of the circuit.

Choke coil :

(i) We know that in purely resistive circuit, the power loss is maximum because power factor $\cos\phi = (R / Z) = 1 \; (\because Z = R)$. Hence, the use of resistance is avoided in AC circuits to control current.

(ii) A choke coil is a coil which has high inductance and negligible resistance. Thus, the power factor is almost zero. So a choke coil controls the alternating current without an apreciable energy loss. This is used with fluorescent tubes to control the current.